Advanced Metamorphosis Based on Bounded Space-time Blending

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Abstract

We further develop a new approach to shape metamorphosis using bounded blending operations in space-time. The key steps of the metamorphosis algorithm are: dimension increase by converting two input kD shapes into half-cylinders in (k+1)D space-time, applying bounded blending union with added material to the half-cylinders, and making cross-sections for getting intermediate shapes under the transformation. This approach is extended here in two directions. First, the problem of “jump” in animation or the rapid transition between shapes in the given interval is solved using “smoothed” versions of half-cylinders which undergo bounded blending. Second, the approach is extended to 3D initial and final shapes with the bounded blending union operation applied to the corresponding “smoothed” 4D space-time half-cylinders.

1. Introduction

General transformations between given shapes in animation and free-form modeling include simple linear transformations (translation, scaling, rotation), non-linear transformations such as free-form transformations and other non-linear space mappings, and metamorphosis or morphing (transformation of one given shape into another). The specific aspects of the shape transformation problem considered in this paper are the following. The initial shapes can have arbitrary topology not corresponding to each other. No restrictions should be imposed on the input shape model, the shapes can be defined as 2D polygons, implicit surfaces, or constructive solids in 2D or 3D. We do not require the shapes to be aligned or overlap, in fact, they can occupy different positions in space. The one-to-one correspondence established between the boundary points or other shape features is not required. We consider a combined transformation including metamorphosis and nonlinear motion.

A brief survey of existing approaches to shape metamorphosis is given in the next section. Implicit surfaces [2] and FRep solids [7] seem to be most suitable for the given task. In this work, we further develop the new method of shape metamorphosis proposed in [9], which is based on increasing the object dimension, function-based bounded blending, and consequent cross-sectioning for animation. Here, we extend this approach in two directions. First, the problem of rapid transition between shapes in the given interval is solved using “smoothed” versions of half-cylinders which undergo bounded blending. Second, the approach is extended to 3D initial and final shapes and bounded blending union between corresponding 4D space-time half-cylinders. Examples of 2D and 3D metamorphosis using the proposed approach are given.

2. Other works

The approaches to 2D metamorphosis include physically-based methods [11,12], star-skeleton representation [13], warping and distance field interpolation [3], wavelet-based [17], and surface reconstruction methods [14]. A detailed survey on 3D metamorphosis can be found in [6]. The existing approaches are based on one or several of the following assumptions: equivalent topology (mainly topological disks or balls are considered), polygonal shape representation, shape alignment (shapes have common coordinate origin and significantly overlap in most of the case studies), possibility of shape matching (establishing of shape vertex-vertex, control points or other features correspondence), the resulting transformation should be close to the motion of an articulated figure.

The desired type of behavior can be obtained using skeletal implicit surfaces [16] with manual or automatic establishing of correspondence between scalar field source points. Metamorphosis of more general shapes based on skeletal implicit surfaces can be handled using...
hierarchical tree structures similar to those used in FRep. Galin et al. [5] proposed to automatically establish correspondence between such tree structures of the source and the target objects with additional manual control from the animator. Metamorphosis of arbitrary FRep objects can be described using the linear (for two initial solids) or bilinear (for four initial solids) function interpolation [4], but it can produce poor results for not aligned objects with different topology. Turk and O’Brien [15] proposed a more sophisticated approach based on interpolation of surface points (with assigned time coordinates) using radial bases functions in 4D space. This method is applicable to not aligned surfaces with different topology. However, for the initially given implicit surfaces this requires time consuming surface sampling and interpolation steps.

3. Metamorphosis using bounded blending

A blending operation in shape modeling generates smooth transition between two curves or surfaces. Note that sometimes the term “blending” is used to designate metamorphosis of 2D shapes, but we use it here in the way traditional to geometric and solid modeling. Blending versions of set-theoretic operations (intersection, union, and difference) on solids approximate exact results of these operations by rounding sharp edges and vertices. Such operations are usually used in computer-aided design for modeling fillets and chamfers. In the case of blending union of two disjoint solids with added material, a single resulting solid with a smooth surface can be obtained. This property of the blending union operation is the basis of our approach to the shape metamorphosis.

Figure 1. Initial 2D shape (union of two disks) and final 2D shape (cross) for metamorphosis.

Figure 2. Two half-cylinders with the given 2D shapes as cross-sections

Let us illustrate step by step the approach we proposed in [9] (Figs. 1-4):

1) two initial shapes are given on the xy-plane (union of two disks and a cross in Fig. 1);
2) each shape is considered as a cross-sections of a half-cylinder in 3D space (a cylinder bounded by a plane from one side) as it is shown in Fig. 2;
3) the axes of both cylinders are parallel to some common straight line in 3D space, for example, to the coordinate z-axis, and the bounding planes of two half-cylinders are placed at some distance to give space for making the blend;
4) apply the added material bounded blending union operation to the half-cylinders (Fig. 3);
5) adjust parameters of the blend such that a satisfactory intermediate 2D shape is obtained in one or several 2D cross sections by planes orthogonal to z-axis (Fig. 4);
6) considering additional z-coordinate as time, make consequent orthogonal cross-sections along z-axis (Fig. 4) and combine them into 2D animation.
The following definition of the bounded blending set-theoretic operation is used:

\[ F_{bb}(f_1, f_2, f_3) = R(f_1, f_2) + \text{disp}_{bb}(f_1, f_2, f_3), \]

(1)

where \( R(f_1, f_2) \) is an \( R \)-function corresponding to the type of the set-theoretic operation [10, 7], the arguments of the operation \( f_1(X) \) and \( f_2(X) \) are defining functions of two initial solids, and \( \text{disp}_{bb}(f_1, f_2, f_3) \) is a displacement function depending on the defining function of the bounding solid \( f_3(X) \).

The formulation for blending operations with the blend bounded by an additional bounding solid was used as proposed in [8]. An appropriate displacement function can be taken in the form:

\[ \text{disp}_{bb}(r) = \begin{cases} (1-r^2)^3, & r < 1 \\ 1+r^2, & r \geq 1 \end{cases} \]

(2)

with \( r^2 = \frac{r_1^2}{r_1^2 + r_2^2}, r_2 > 0 \),

(3)

where

\[ r_1^2(f_1, f_2) = \left( \frac{f_1}{a_1} \right)^2 + \left( \frac{f_2}{a_2} \right)^2, \]

and

\[ r_2^2(f_3) = \begin{cases} \left( \frac{f_1}{a_3} \right)^2, & f_3 > 0 \\ 0, & f_3 \leq 0 \end{cases} \]

with numerical parameters \( a_1 \) and \( a_2 \) controlling the blend symmetry, and \( a_3 \) allowing the user to interactively control the influence of the function \( f_3 \) on the overall shape of the blend. This definition of the function \( r \) and the definition (2) of the displacement function \( \text{disp}_{bb}(r) \) are not unique and can be changed, if it is necessary in particular applications.

The application of the bounded blending union to the half-cylinders for the shape metamorphosis is illustrated by Figs. 3 and 4. The half-cylinders are bounded by the planes \( z=0 \) and \( z=1 \) to make the gap \([0,1]\) along \( z \)-axis.
between them. The bounding solid for the blend in this case is an infinite slab orthogonal to $z$-axis and defined by the function $f_3$ as an intersection of two halfspaces with the definitions $z \geq -10$ and $z \leq 10$. The blending displacement from the exact union of two half-cylinders takes zero value at the boundaries of the bounding solid (planes $z=-10$ and $z=10$). This results in the exact initial 2D shapes obtained at the cross-sections outside the bounding solid: two disks for $z \leq -10$ and the cross for $z \geq 10$ (first and last images in Fig. 4 respectively). The parameters $a_0 - a_3$ of the bounded blend influence the blend shape and respectively the shape of the intermediate cross-sections. Note that whatever interval is selected for the bounded blend along $z$-axis in 3D space, it can always be scaled to match the required time interval for the shape transformation on a 2D plane.

4. Algorithm extensions

The main problem with the bounded blending (Fig. 3) and resulting animation (Fig. 4) is that the most significant part of the shape transformation happens in the $[0,1]$ interval with the 2D shape changing rapidly from the initial to the final cross-section, which results in the visible “jump” in animation during this time interval. The main reason is that the bounded blending is applied to a half-cylinder bounded by a plane orthogonal to the axis. This set-theoretic subtraction of one cylinder half results in the sharp edge of the half-cylinder boundary (as seen in Fig. 2) with this edge remaining a significant feature of the blended half-cylinders (see edges at the top and bottom parts of the shape in Fig. 3 left).

To avoid the described problem of the “jump” in the animation or of the rapid transition between shapes in the given interval, we propose in 3.1 to use “smoothed” versions of half-cylinders which undergo bounded blending. Then, in 3.2 the general approach is extended to 3D initial and final shapes and the bounded blending union operation is applied to corresponding “smoothed” 4D space-time half-cylinders.

3.1 Smoothing half-cylinders

To avoid the sharp edges in the initial half-cylinders and in the resulting bounded blending, we propose to apply more “smooth” operation between the cylinder and the bounding planar half-space. The pure set-theoretic subtraction resulting in the sharp edge can be replaced by the bounded blending subtraction.

For example, the half-cylinder with the cross shape (Fig. 2 right) was generated by the pure subtraction of the halfspace $z \leq 1$, which instead can be replaced by the bounded blending subtraction based on equations (1) and (2), where $f_1(X)$ defines an infinite cylinder, $f_2(x, y, z) = 1 - z$ is subtracted from the cylinder, and $f_3(x, y, z) = 5 - z$ is the bounding solid for the blended subtraction, which defines the area of “smoothing”.

The resulting shape of the “smoothed” half-cylinder is shown in Fig. 5 (top-right). Fig. 5 also shows possible shapes of the “smoothed” half-cylinders depending on different parameters of the bounded blending subtraction.
7) consequent orthogonal cross-sections along the time axis are made and combined into a 3D animation.

Figure 7. Frames of animation based on the bounded blending between “smoothed” half-cylinders: no “jump” in animation is observed.

3.2 3D metamorphosis using space-time blending

As no assumptions were made in the proposed approach about the dimensionality of the initial shapes, we can apply it to 3D objects. The bounded space-time blending procedure for initial 3D shapes consists of the following steps analogous to those applied for 2D shapes and illustrated in Fig. 8:

1) two initial 3D shapes are given in xyz-space (see the initial cube and the union of two tori in Fig. 8);
2) each shape is considered as a 3D cross-section of a half-cylinder defined in 4D space-time (a cylinder bounded by a plane from one side along the time axis);
3) the bounding planes of two half-cylinders are placed at some distance along time axis to provide a time interval for making the blend;
4) the 4D half-cylinders are smoothed using the bounded blending subtraction of the planar half-spaces;
5) the added material bounded blending union operation is applied to the “smoothed” 4D half-cylinders;
6) parameters of the blending union are adjusted such that satisfactory intermediate 3D shapes are obtained in one or several cross sections along the time axis (see Fig. 8 showing four intermediate shapes);
7) consequent orthogonal cross-sections along the time axis are made and combined into a 3D animation.

Note that similar to the 2D case, topological changes of 3D objects are handled automatically. The initial cube in Fig. 8 has genus 0, while the final object has genus 4.

Figure 8. Metamorphosis of a cube into the union of two tori

Some unwanted disconnected components can appear during the metamorphosis (as in Fig. 8 middle-left). Let us now try to fine tune the process by adding user controlled deformations. The appearance of the disconnected component in Fig. 8 can be explained by quite big distance between the initial cube and the final union of two tori. We can improve the metamorphosis by adding time-dependent deformation of the cube in the direction of the tori’s center. This can be done with help of a non-linear space mapping (“warping”) controlled by a single point attached to the front face of the cube and moved towards the tori’s center. The balance between the deformation and the metamorphosis can be found by selecting appropriate time schedules for both processes. In Fig. 9, we can observe that the cube is only deformed at
the beginning (upper-right frame) and the actual metamorphosis starts later to avoid the possibility for disconnected components to appear.

Figure 9. Metamorphosis of a cube into the union of two tori improved by applying additional deformation.

5. Conclusion

We have further developed the new approach to shape metamorphosis on the basis of the bounded space-time blending between higher-dimensional objects. The proposed approach can handle non-overlapping 2D and 3D shapes with arbitrary topology. The obtained behavior during the transformation process does not imitate the motion of an articulated figure, but rather has amorphous or amoeba-like character including non-linear motion and metamorphosis. In this paper, we extended the proposed approach in two directions: additional control of the metamorphosis is introduced using “smooth” half-cylinders in space-time, and the metamorphosis between 3D shapes is described and tested.

There are several issues requiring further research and development in the proposed direction. The user control of the entire process including elimination of unwanted disconnected components will be further investigated. Although the topological changes are generated automatically, analytical or numerical analysis of the transformation is needed to extract the critical points along the time axis, which can be useful for generation of more representative animation and in other applications.

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References


